NORMANHURST BOYS HIGH SCHOOL

MATHEMATICS ADVANCED

2022 Year 12 Course Assessment Task 4 (Trial HSC Examination)
Thursday, 11 August 2022

General instructions

- Working time – 3 hours.
  (plus 10 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- Attempt **all** questions.

SECTION I

- Mark your answers on the answer grid provided (on page 29)

SECTION II

- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

NESA STUDENT #: ..... **# BOOKLETS USED**: .....

Class (please ✔)

- ◯ 12MAA.1 – Miss J. Kim
- ◯ 12MAA.2 – Mr Ho
- ◯ 12MAX.1 – Mr Lam
- ◯ 12MAX.2 – Miss C. Kim
- ◯ 12MAX.3 – Miss Lee

Marker's use only.

QUESTION	1-10	11-15	16-19	20-21	22-26	27-30	31-33	Total
MARKS	/10	/12	/16	/11	/18	/17	/16	/100

Section I

10 marks
Attempt Question 1 to 10
Allow approximately 15 minutes for this section

Mark your answers on the answer grid provided (labelled as page 29).

Questions **Marks**

1. A circle has the equation $(x+3)^2+(y-4)^2=d$. What is the value of d such that the x-axis is a tangent to the circle? **1**

(A) $d=9$ (B) $d=3$ (C) $d=4$ (D) $d=16$

2. What is the domain of the function $f(x)=\ln(3-x)+\sqrt{5x-4}$? **1**

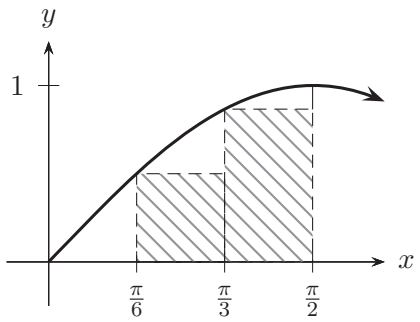
(A) $x\in\left(\frac{4}{5},3\right)$ (B) $x\in\left(\frac{4}{5},3\right]$ (C) $x\in\left[\frac{4}{5},3\right)$ (D) $x\in\left[\frac{4}{5},3\right]$

3. A particle, initially 3 m to the right of the origin has a velocity given by $v=(7-3t)$ ms^{-1}. What is an expression for the the displacement of the particle? **1**

(A) $x=7t-\dfrac{3t^2}{2}+3$ (C) $x=7t-3t^2+3$

(B) $x=7t-\dfrac{3t^2}{2}+C$ (D) $x=7t-3t^2$

4. The area beneath the curve $y=\sin x$ between $x=0$ and $x=\frac{\pi}{2}$ is approximated by the two rectangles as shown. **1**



What is the approximation to the area?

(A) $\dfrac{\pi}{2}$ square units (C) $\dfrac{\left(1+2\sqrt{3}\right)\pi}{12}$ square units

(B) $\dfrac{2\pi}{3}$ square units (D) $\dfrac{\left(1+\sqrt{3}\right)\pi}{12}$ square units

5. Which of the following is an expression for $\dfrac{d}{dx}\left(4\tan\left(\dfrac{5\pi}{3}x\right)\right)$? **1**

(A) $\dfrac{20\pi}{3}\sec\left(\dfrac{5\pi}{3}x\right)$

(B) $\dfrac{20\pi}{3}\sec^2\left(\dfrac{5\pi}{3}x\right)$

(C) $4\sec\left(\dfrac{5\pi}{3}x\right)$

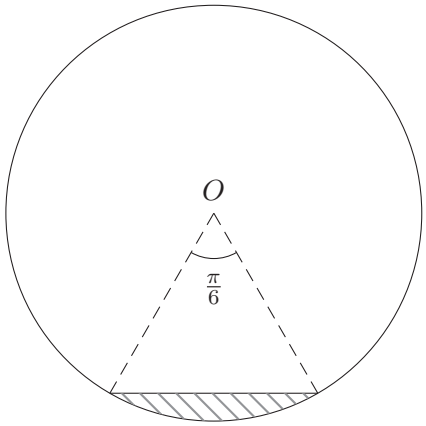
(D) $4\sec^2\left(\dfrac{5\pi}{3}x\right)$

6. Consider the two sets $A = \{6, 7, 8\}$ and $B = \{11, 12, 13\}$. A number is randomly selected from each set. **1**

What is the probability of selecting a number from each set that has a sum equal to or greater than 20?

(A) $\dfrac{1}{9}$ (B) $\dfrac{2}{9}$ (C) $\dfrac{1}{3}$ (D) $\dfrac{1}{6}$

7. The area of a minor segment of the circle pictured below is 100π square metres. **1**



NOT TO SCALE

Which of the following is the exact form of the radius of the circle?

(A) $\sqrt{\dfrac{200\pi(\pi-3)}{6}}$

(B) $\sqrt{\dfrac{1200}{\pi-3}}$

(C) $\sqrt{\dfrac{200(\pi-3)}{\pi}}$

(D) $\sqrt{\dfrac{1200\pi}{\pi-3}}$

Examination continues overleaf...

8. Two hundred people were tested for COVID with the following results. **1**

	Tests positive	**Tests negative**
Showed symptoms	96	24
Did not show symptoms	8	72

What is the probability that the person tests positive to COVID, given that they didn't show symptoms?

(A) $\frac{1}{10}$ (B) $\frac{4}{5}$ (C) $\frac{11}{13}$ (D) $\frac{12}{13}$

9. The probability distribution of a discrete random variable X is shown below. **1**

x	1	2	3	4	5	6
$P(X = x)$	0.12	$\frac{a}{2}$	a	0.35	0.07	a

Which of the following is the value of a?

(A) 0.54 (C) 0.184

(B) 0.46 (D) 0.216

10. The displacement x metres of a particle moving in a straight line at time t seconds is given by: **1**

$$x = 2t - 4\log_e(2t + 1)$$

Which of the following statements is **false**?

(A) The acceleration is always positive (C) The particle is initially at the origin

(B) The initial velocity is $-6\,\text{ms}^{-1}$ (D) As $t \to \infty$, $x \to \infty$ and $v \to \infty$.

Examination continues overleaf...

Section II

90 marks
Attempt Question 11 to 33
Allow approximately 2 hours and 45 minutes for this section

Write your answers in the space provided.

Question 11 (2 marks)

Given the functions $f(x) = 2x^2 - 7x + 5$ and $g(x) = 3x + 2$, find $f\Big(g(x)\Big)$ in simplest form. **2**

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Question 12 (2 marks)

Find the integers a and b such that $\left(5 - \sqrt{3}\right)^2 = a - b\sqrt{3}$ **2**

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Question 13 (2 marks)

Fully simplify $\sqrt{45} + 2\sqrt{5} - \sqrt{90}$ **2**

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Question 14 (2 marks)

Solve $|3x - 7| = 5$. **2**

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Examination continues overleaf...

Question 15 (4 marks)

(a) Carefully state the transformations that need to be applied to $y = \dfrac{1}{x}$ to obtain $y = \dfrac{3}{2x+4} - 1$. **2**

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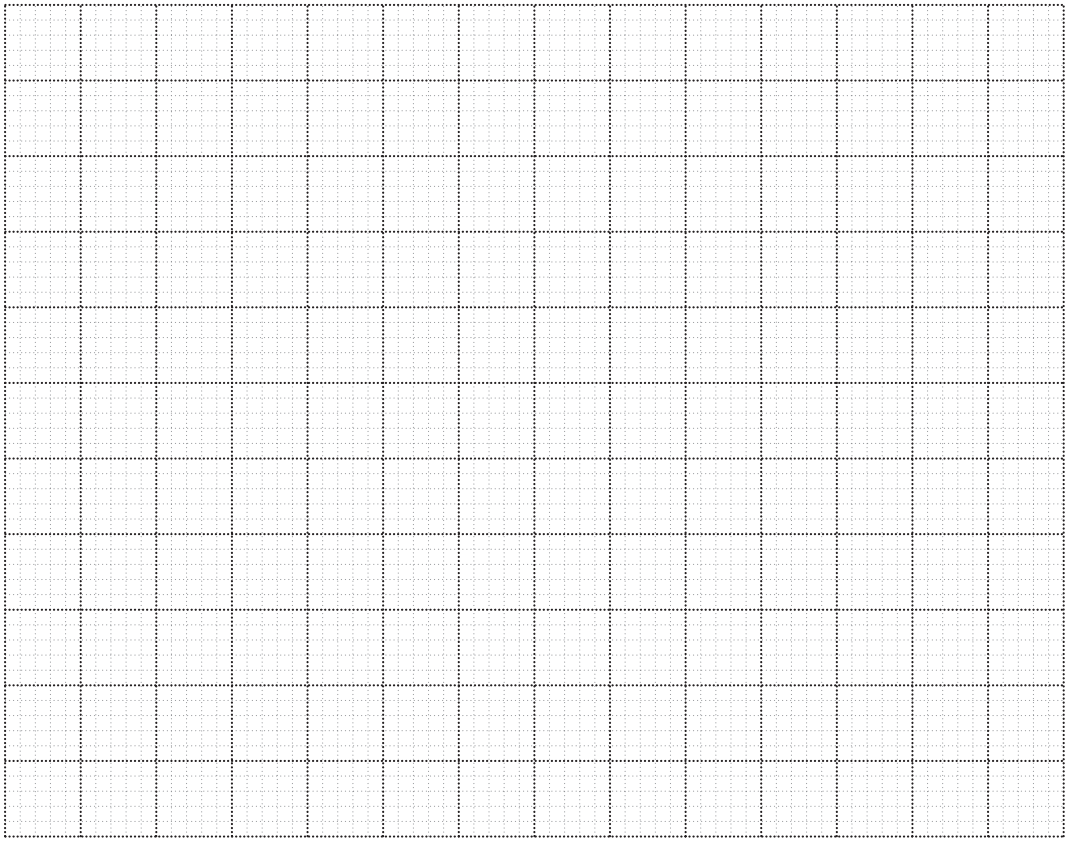
(b) Hence, draw a neat sketch of $y = \dfrac{3}{2x+4} - 1$ **2**



Examination continues overleaf...

Question 16 (2 marks)

Solve the equation **2**

$$4\cos x + 2\sqrt{3} = 0$$

where $0 \leq x \leq 2\pi$.

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Question 17 (3 marks)

Prove that $\dfrac{\sec\theta - \sec\theta\cos^4\theta}{1+\cos^2\theta} = \sin\theta\tan\theta$ **3**

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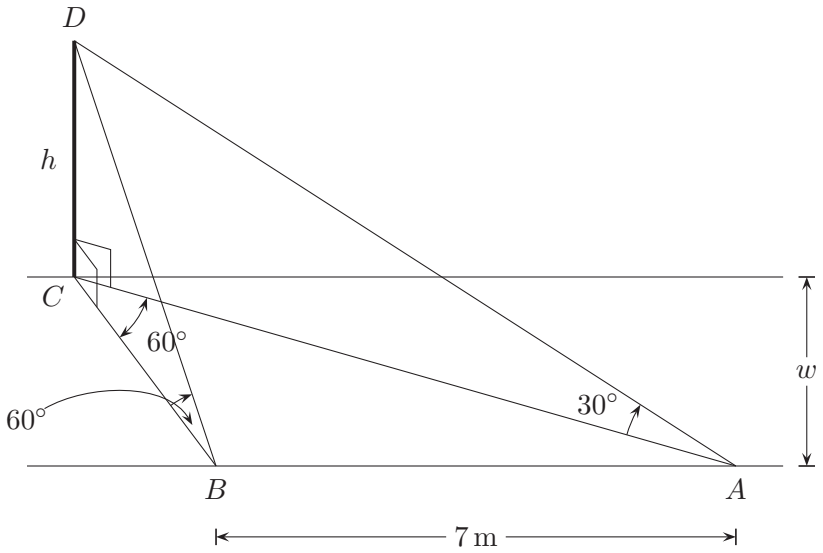
Examination continues overleaf...

Question 18 (6 marks)

A footpath on horizontal ground has two parallel edges. CD is a vertical street sign pole of height h metres which stands with its base C on one edge of the footpath. A and B are two points on the other edge of the footpath such that $AB = 7\,\text{m}$ and $\angle ACB = 60°$. From A and B, the angles of elevation to the top of the pole at D are $30°$ and $60°$ respectively.



(a) Show that the exact height of the flagpole is $h = \sqrt{21}$ metres. **3**

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(b) By considering the area of $\triangle ABC$, find the exact width w of the footpath. **3**

Examination continues overleaf...

Question 19 (5 marks)

An office has climate control and the temperature at any time t hours is T degrees Celsius, where

$$T(t) = 22 + 3\sin\left(\frac{\pi}{12}t\right) \qquad t \in [0, 24]$$

(a) What are the minimum and maximum temperatures in this office? **2**

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(b) Miss Kim brought her lunch into the office but forgot to place it in the refrigerator. The food will be spoilt if the surrounding room temperature is greater than 23.5°C for more than 3 hours. **3**

Let $t = 0$ correspond to the time which Miss Kim arrives at the office (8:00 am). What is the latest time that she should finish eating the lunch, before the food is spoilt?

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Question 20 (5 marks)

Thirty Transit Officers patrolled the train line between Strathfield and Hornsby for three days. The number of ticket infringements issued by each Transit Officer over their three day blitz is summarised in the following table.

Number of ticket infringements	Frequency
0	12
1	5
2	6
3	3
4	3
5	1

(a) Calculate the mean number of infringements issued over the past three days. Give your answer correct to two decimal places. **1**

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(b) Calculate the standard deviation. Give your answer correct to two decimal places. **1**

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(c) Determine the median number of infringements issued and describe the skewness of the data. **2**

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(d) There are 120 Transit Officers patrolling the rail network. Use the sample data in the table to estimate how many of these Transit Officer would have issued more than two ticketing infringements within their three day blitz. **1**

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Examination continues overleaf...

Question 21 (6 marks)

(a) The September sale prices for properties (in dollars) in a suburb sold by a particular real estate agent were: **2**

747 000, 768 000, 887 000, 895 000, 959 000, and 986 000

On the 1st of October they realised they had omitted a sale price of 992 000.

If this sale price was included in the September summary, what effect would it have on the mean and median?

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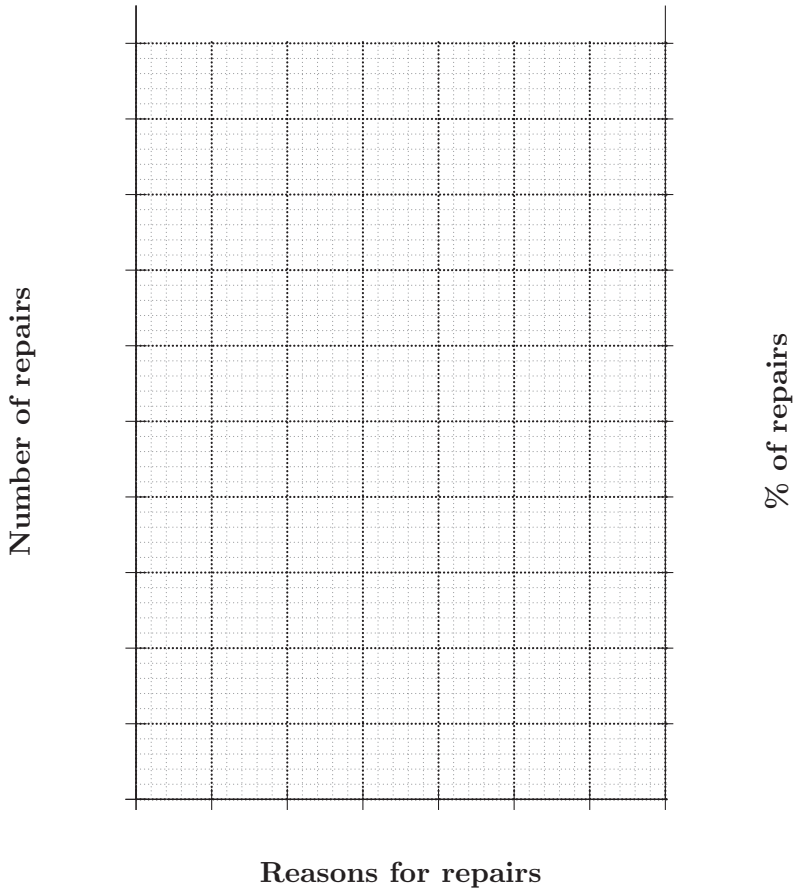
(b) A computer repair store summarised the repair inquiries they received in one month in the following table:

Type of repair	**Number of inquiries**	**Cumulative percentage**
Screen	360	
Malfunctioning trackpad	210	
Malfunctioning key on keyboard	160	
Damaged USB Port	110	
Battery replacement	90	
Overheating	70	

i. Fill in the table's **cumulative percentage** column and construct the Pareto chart for this information. **2**



ii. Which computer problems account for 70% of the inquiries? Give a reason for why this may be the case. **2**

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Examination continues overleaf...

Question 22 (4 marks)

Differentiate the following:

(a) $\ln(\tan 3x)$ **2**

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(b) $\dfrac{x}{e^{3x+1}}$ **2**

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Question 23 (3 marks)

Find the equation of the normal to the curve $y = \log_e\left(2x^2+1\right)$ at the point $(2, \log_e 9)$. **3**

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Examination continues overleaf...

Question 24 (4 marks)

(a) Find: $\int \left(1 + e^{7x}\right) dx$ **2**

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(b) Evaluate: $\int_0^2 \frac{2}{5x+4} dx$ **2**

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Question 25 (4 marks)

(a) Find $\frac{d}{dx}\left(xe^{2x}\right)$. **2**

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(b) Hence or otherwise, find **2**

$$\int xe^{2x}\, dx$$

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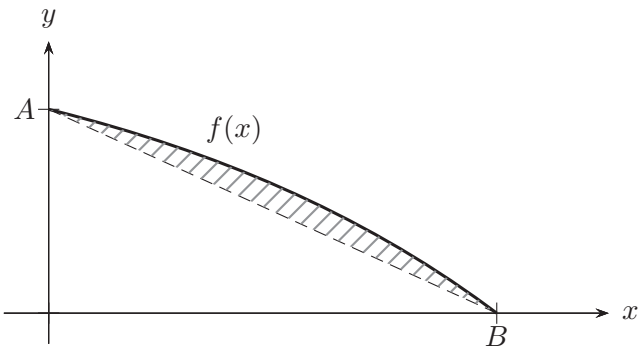
Examination continues overleaf...

Question 26 (3 marks)

Part of the graph of the function $f(x) = 9 - 3^{x+1}$ is shown below. **3**



$f(x)$ intersects the axes at the points A and B. The line joining A and B is also shown.

Show that the exact value of the shaded area is

$$6 - \frac{6}{\ln 3}$$

Question 27 (2 marks)

A sports magazine surveyed 120 soccer fans and asked them whether they supported Manchester United or Tottenham. 74 fans supported Manchester United, 55 supported Tottenham and 8 supported neither team.

(a) Represent this data in a Venn diagram in the space below. **1**

(b) A fan is selected at random. If the fan selected is a Manchester United fan, what is the probability that they are also a Tottenham fan? **1**

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Question 28 (5 marks)

Elysia and Leo are playing a game, they take turns throwing two dice. The game is won by the first player to throw a double six. Elysia starts the game.

(a) What is the probability that Elysia wins the game on the first throw. **1**

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Examination continues overleaf...

(b) What is the probability that Elysia wins the game on the first or on the second throw? **2**

(c) Find the probability that Elysia eventually wins the game. **2**

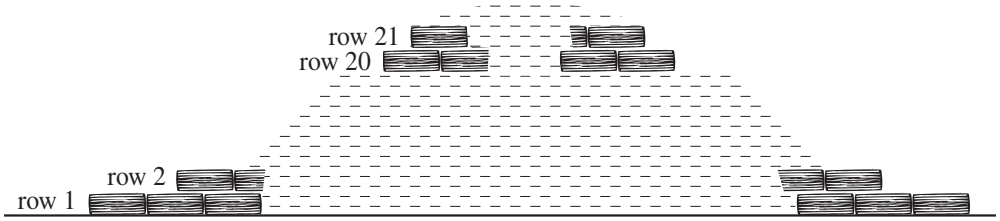
Examination continues overleaf...

Question 29 (5 marks)

Mr Park has built a new brick wall in his backyard to patch up the hole that had been created when a car ran into his fence.

The bottom row has 75 bricks, and each of the rows 2, 3, ..., 20 has 3 fewer bricks than the row below it.

Above row 20, each row has 1 brick fewer than the row below it. The top row has 10 bricks.



(a) How many bricks are in row 20? **2**

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(b) What is the total number of rows in the wall? **1**

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(c) How many bricks are used in the construction of the wall? **2**

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Examination continues overleaf...

Question 30 (5 marks)

Scientists have recently discovered a new drug to control a rare medical condition. It is known that the quantity Q of the drug remaining in the body after t hours satisfies an equation of the form

$$Q = Q_0 e^{-kt}$$

where Q_0 and k are positive constants.

The initial dose is 5 milligrams and after 17 hours the amount remaining in the body is half the initial dose.

(a) Find the exact values of Q_0 and k. **3**

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(b) When will one-seventh of the initial dose remain? **2**

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Examination continues overleaf...

Question 31 (5 marks)

A particle with velocity, v ms^{-1} moves along a straight line from a fixed point O

$$v = -4\cos t$$

where the time t is measured in seconds from $t = 0$.
It is initially 1 metre to the right of the origin.

(a) When, and where, does the particle first come to rest? **2**

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(b) Show that the total distance travelled by the particle between $0 \leq t \leq \dfrac{2\pi}{3}$ is $(8 - 2\sqrt{3})$ metres. **3**

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Examination continues overleaf...

Question 32 (6 marks)

A ride sharing vehicle is to complete an interstate trip totalling 1000 kilometres at a constant speed of v km/h. When travelling at this speed the car consumes fuel at the rate of

$$\left(6 + \frac{v^2}{50}\right) \text{ litres per hour.}$$

As a bonus, the company has offered to pay 50 cents per litre for fuel and for safety, two drivers where they will each be paid 45 dollars per hour whilst they are completing the trip.

(a) Let C represent the total cost of fuel and the drivers' wages for the trip. **3**

Show that

$$C = 10v + \frac{93000}{v}$$

if the company was to pay two drivers.

(b) The vehicle needs to complete the 1000 kilometre journey within 12 hours, whilst remaining under the speed limit of 90 km/h. **3**

At what speed v should the vehicle be driven at to minimise the cost C, whilst arriving at the destination within 12 hours?

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Examination continues overleaf...

Question 33 (5 marks)

For the function $f(x) = x^3 - 3x^2 - 9x + 18$, find the stationary points, point(s) of inflection and determine the nature of the stationary points. Then sketch the graph of $f(x)$, showing the turning points and point(s) of inflection. **5**

End of paper.

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  (plus 10 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- Attempt **all** questions.

SECTION I

- Mark your answers on the answer grid provided (on page 29)

SECTION II

- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

NESA STUDENT #: ..Handwritten Solutions..... **# BOOKLETS USED:** .....

Class (please ✔)

- ◯ 12MAA.1 – Miss J. Kim
- ◯ 12MAA.2 – Mr Ho
- ◯ 12MAX.1 – Mr Lam
- ◯ 12MAX.2 – Miss C. Kim
- ◯ 12MAX.3 – Miss Lee

Marker's use only.

QUESTION	1-10	11-15	16-19	20-21	22-26	27-30	31-33	Total
MARKS	/10	/12	/16	/11	/18	/17	/16	/100

Answer sheet for Section I

Mark answers to Section I by fully blackening the correct circle, e.g "●"

NESA STUDENT #: .....

Class (please ✔)

◯ 12MAA.1 – Miss J. Kim

◯ 12MAA.2 – Mr Ho

◯ 12MAX.1 – Mr Lam

◯ 12MAX.2 – Miss C. Kim

◯ 12MAX.3 – Miss Lee

Directions for multiple choice answers

- Read each question and its suggested answers.
- Select the alternative Ⓐ, Ⓑ, Ⓒ, or Ⓓ that best answers the question.
- Mark only one circle per question. There is only *one* correct choice per question.
- Fill in the response circle completely, using blue or black pen, e.g.

Ⓐ Ⓑ ● Ⓓ

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

Ⓐ Ⓑ ✕● ●

- If you continue to change your mind, write the word `correct` and clearly indicate your final choice with an arrow as shown below:

Ⓐ Ⓑ ✕● ●← correct

1 – Ⓐ Ⓑ Ⓒ **Ⓓ**
2 – Ⓐ Ⓑ **Ⓒ** Ⓓ
3 – **Ⓐ** Ⓑ Ⓒ Ⓓ
4 – Ⓐ Ⓑ Ⓒ **Ⓓ**
5 – Ⓐ **Ⓑ** Ⓒ Ⓓ

6 – Ⓐ Ⓑ **Ⓒ** Ⓓ
7 – Ⓐ Ⓑ Ⓒ **Ⓓ**
8 – **Ⓐ** Ⓑ Ⓒ Ⓓ
9 – Ⓐ Ⓑ **Ⓒ** Ⓓ
10 – Ⓐ Ⓑ Ⓒ **Ⓓ**

Section II

90 marks
Attempt Question 11 to 33
Allow approximately 2 hours and 45 minutes for this section

Write your answers in the space provided.

Question 11 (2 marks)

Given the functions $f(x) = 2x^2 - 7x + 5$ and $g(x) = 3x + 2$, find $f\Big(g(x)\Big)$ in simplest form. **2**

$f(g(x)) = 2(3x+2)^2 - 7(3x+2) + 5$ (1)

$= 2(9x^2 + 12x + 4) - 7(3x+2) + 5$

$= 18x^2 + 24x + 8 - 21x - 14 + 5$

$= 18x^2 + 3x - 1$ (1)

Question 12 (2 marks)

Find the integers a and b such that $\left(5 - \sqrt{3}\right)^2 = a - b\sqrt{3}$ **2**

$(5 - \sqrt{3})^2 = 25 - 10\sqrt{3} + 3$

$= 28 - 10\sqrt{3}$

$\therefore a = 28, \quad b = 10$ (2) correct value of a and b

Question 13 (2 marks)

Fully simplify $\sqrt{45} + 2\sqrt{5} - \sqrt{90}$ **2**

$\sqrt{9 \times 5} + 2\sqrt{5} - \sqrt{9 \times 10} = 3\sqrt{5} + 2\sqrt{5} - 3\sqrt{10}$ (1)

$= 5\sqrt{5} - 3\sqrt{10}$ (1)

Question 14 (2 marks)

Solve $|3x - 7| = 5$. **2**

$3x - 7 = 5 \qquad -3x + 7 = 5$

$3x = 12 \qquad -3x = -2 \qquad \therefore x = 4 \text{ or } \frac{2}{3}$

$x = 4$ (1) $\qquad x = \frac{2}{3}$ (1)

Examination continues overleaf...

Question 15 (4 marks)

(a) Carefully state the transformations that need to be applied to $y = \dfrac{1}{x}$ to obtain $y = \dfrac{3}{2x+4} - 1$. **2**

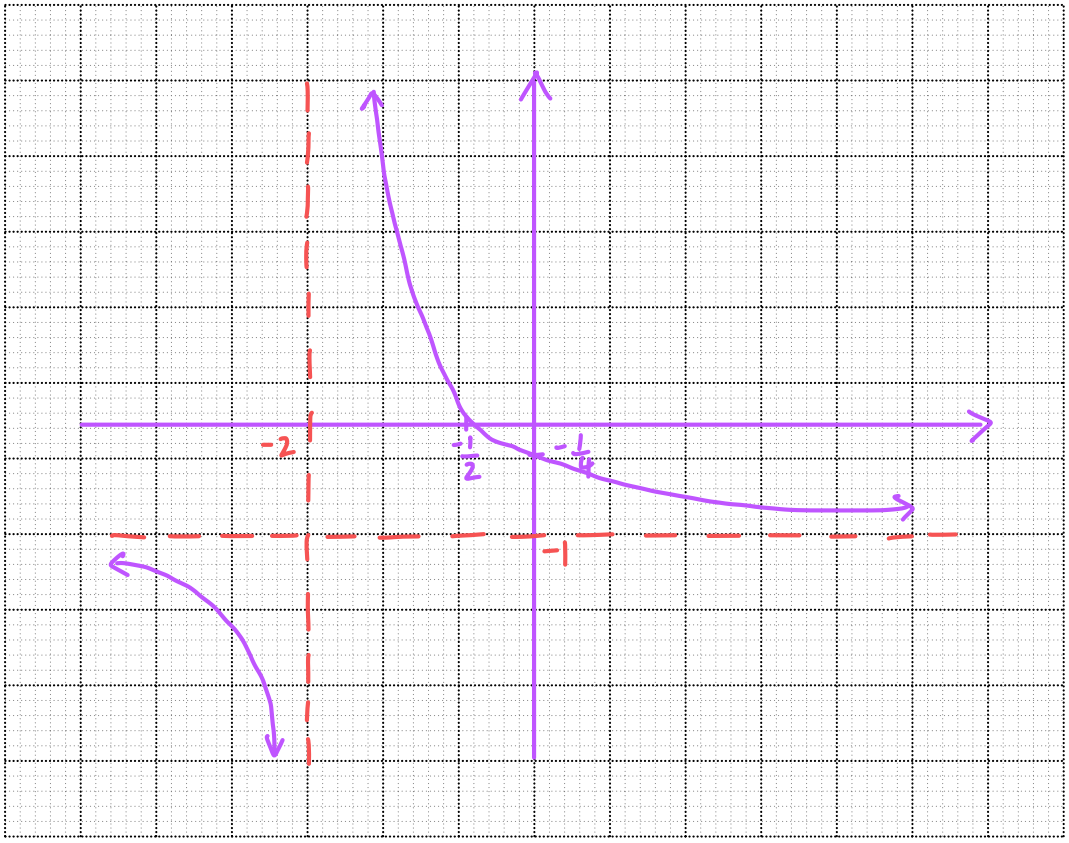
$y = \dfrac{3}{2(x+2)} - 1$

- Horizontal dilation by $\frac{1}{2}$
- Shift left by 2
- Vertical dilation by 3
- Shift down by 1

① correct dilations

① correct translations

(b) Hence, draw a neat sketch of $y = \dfrac{3}{2x+4} - 1$ **2**



$1 = \dfrac{3}{2x+4}$

$2x+4 = 3$

$2x = -1$

$x = -\frac{1}{2}$

① correct intercepts & asymptotes

① hyperbola

Examination continues overleaf...

Question 16 (2 marks)

Solve the equation **2**

$$4\cos x + 2\sqrt{3} = 0$$

where $0 \leq x \leq 2\pi$.

$4\cos x = -2\sqrt{3}$

$\cos x = -\frac{\sqrt{3}}{2}$ (1) reference $\angle = \frac{\pi}{6}$

$x = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}$ $\therefore$ $x = \frac{5\pi}{6}, \frac{7\pi}{6}$ (1)

Question 17 (3 marks)

Prove that $\dfrac{\sec\theta - \sec\theta\cos^4\theta}{1+\cos^2\theta} = \sin\theta\tan\theta$ **3**

$$\text{LHS} = \frac{\sec\theta - \sec\theta\cos^4\theta}{1+\cos^2\theta}$$

$$= \frac{\frac{1}{\cos\theta} - \frac{\cos^4\theta}{\cos\theta}}{1+\cos^2\theta}$$ (1)

$$= \frac{1-\cos^4\theta}{\cos\theta(1+\cos^2\theta)}$$

$$= \frac{(1+\cos^2\theta)(1-\cos^2\theta)}{\cos\theta(1+\cos^2\theta)}$$ (1)

$$= \frac{1-\cos^2\theta}{\cos\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta}$$

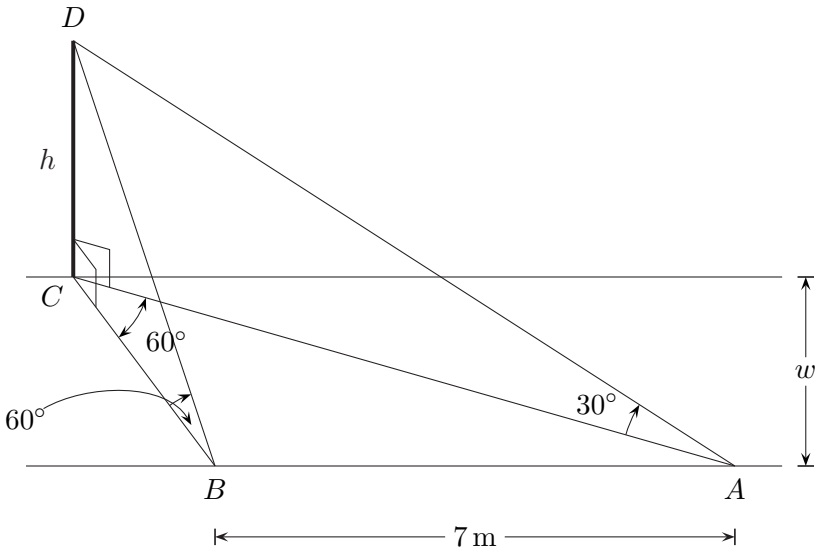
$$= \frac{\sin\theta}{\cos\theta} \times \sin\theta$$ (1)

$$= \sin\theta\tan\theta$$

$$= \text{RHS}$$

Question 18 (6 marks)

A footpath on horizontal ground has two parallel edges. CD is a vertical street sign pole of height h metres which stands with its base C on one edge of the footpath. A and B are two points on the other edge of the footpath such that $AB = 7\,\text{m}$ and $\angle ACB = 60°$. From A and B, the angles of elevation to the top of the pole at D are $30°$ and $60°$ respectively.



(a) Show that the exact height of the flagpole is $h = \sqrt{21}$ metres. **3**

$\tan 60° = \frac{h}{CB}$ $\tan 30° = \frac{h}{CA}$

$CB = \frac{h}{\tan 60°}$ $CA = \frac{h}{\tan 30°} = \frac{h}{\frac{1}{\sqrt{3}}}$

$= \frac{h}{\sqrt{3}}$ $= \sqrt{3}h$ ①

$AB^2 = CB^2 + CA^2 - 2(CB)(CA)\cos 60°$

$7^2 = \left(\frac{h}{\sqrt{3}}\right)^2 + (\sqrt{3}h)^2 - 2\left(\frac{h}{\sqrt{3}}\right)(\sqrt{3}h)\left(\frac{1}{2}\right)$ ①

$49 = \frac{h^2}{3} + 3h^2 - h^2$

$147 = 7h^2$

$h^2 = 21$

$\therefore h = \pm\sqrt{21}$ (but $h > 0$)

$\therefore h = \sqrt{21}$ metres ①

(b) By considering the area of $\triangle ABC$, find the exact width w of the footpath. **3**

$A = \frac{1}{2}ab\sin C$

$A_{\Delta ABC} = \frac{1}{2}(CB)(CA)\sin C$

$= \frac{1}{2} \times \left(\frac{h}{\sqrt{3}}\right) \times \left(\sqrt{3}h\right) \times \left(\frac{\sqrt{3}}{2}\right)$ (1)

$= \frac{\sqrt{3}}{4}h^2$ where $h = \sqrt{21}$

$= \frac{\sqrt{31}}{4} \times 21$

$= \frac{21}{4}\sqrt{3}$

$A_{\Delta ABC} = \frac{1}{2} \times b \times w$ where $b = 7$

$\frac{21}{4}\sqrt{3} = \frac{1}{2} \times 7 \times w$ (1)

$w = \frac{3\sqrt{3}}{2}$

$\therefore$ width of the footpath is $\frac{3\sqrt{3}}{2}$ metres (1)

Examination continues overleaf...

Question 19 (5 marks)

An office has climate control and the temperature at any time t hours is T degrees Celsius, where

$$T(t) = 22 + 3\sin\left(\frac{\pi}{12}t\right) \qquad t \in [0, 24]$$

(a) What are the minimum and maximum temperatures in this office? **2**

minimum = 22 − 3 = 19° (1)

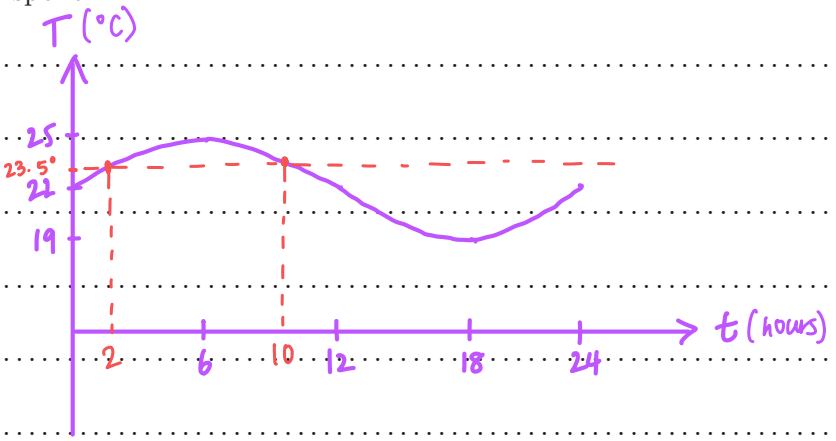
maximum = 22 + 3 = 25° (1)

(b) Miss Kim brought her lunch into the office but forgot to place it in the refrigerator. The food will be spoilt if the surrounding room temperature is greater than 23.5°C for more than 3 hours. **3**

Let $t = 0$ correspond to the time which Miss Kim arrives at the office (8:00 am). What is the latest time that she should finish eating the lunch, before the food is spoilt?

$a = 3$,

$T = \frac{2\pi}{\frac{\pi}{12}} = 24$



When $t = 0$: $T(0) = 22 + 3\sin\left(\frac{\pi}{12}(0)\right)$

$= 22°C$

When $T = 23.5°C$: $23.5 = 22 + 3\sin\left(\frac{\pi}{12}t\right)$ (1)

$\sin\left(\frac{\pi}{12}t\right) = \frac{1}{2}$

$\frac{\pi}{12}t = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$\frac{\pi}{12}t = \frac{\pi}{6}, \frac{5\pi}{6}$

$\therefore t = 2, 10$ (1)

Food is exposed to 23.5°C at 10am.
Therefore, 1pm is the latest time. (1)

Question 20 (5 marks)

Thirty Transit Officers patrolled the train line between Strathfield and Hornsby for three days. The number of ticket infringements issued by each Transit Officer over their three day blitz is summarised in the following table.

Number of ticket infringements	Frequency
0	12
1	5
2	6
3	3
4	3
5	1

(a) Calculate the mean number of infringements issued over the past three days. Give your answer correct to two decimal places. **1**

$\bar{x} = \dfrac{(12\times0)+(5\times1)+(6\times2)+(3\times3)+(3\times4)+(1\times5)}{30}$ (1)

$= 1.43$

(b) Calculate the standard deviation. Give your answer correct to two decimal places. **1**

$\sigma = \dfrac{\Sigma(x_i-\bar{x})^2 f_i}{n}$ (1)

$= 1.4985...$

$= 1.50$

(c) Determine the median number of infringements issued and describe the skewness of the data. **2**

$\frac{30+1}{2} = 15.5$, median $= \frac{1+1}{2} = 1$ (1), positively skewed (1)

(d) There are 120 Transit Officers patrolling the rail network. Use the sample data in the table to estimate how many of these Transit Officer would have issued more than two ticketing infringements within their three day blitz. **1**

$7\times4 = 28$

$\therefore$ 28 transit officers (1)

Examination continues overleaf...

Question 21 (6 marks)

(a) The September sale prices for properties (in dollars) in a suburb sold by a particular real estate agent were: **2**

747 000, 768 000, 887 000, 895 000, 959 000, and 986 000

On the 1st of October they realised they had omitted a sale price of 992 000.

If this sale price was included in the September summary, what effect would it have on the mean and median?

The mean would increase (1) and the median would now change to 895 000. (1)

(b) A computer repair store summarised the repair inquiries they received in one month in the following table:

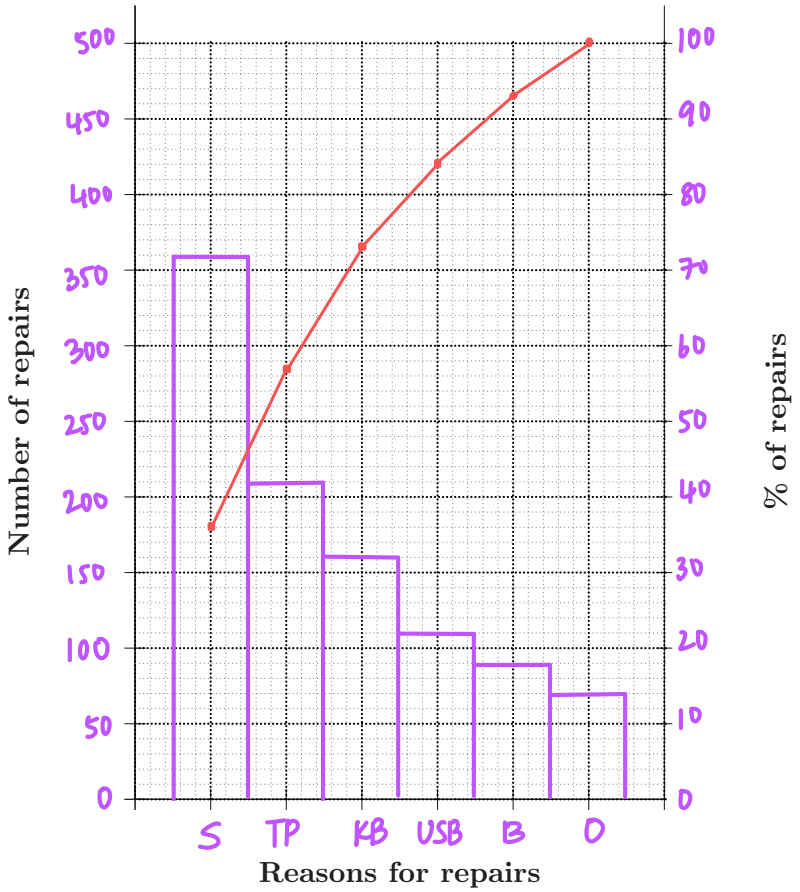
	Type of repair	Number of inquiries	Cumulative percentage
(S)	Screen	360	36%
(TP)	Malfunctioning trackpad	210	57%
(KB)	Malfunctioning key on keyboard	160	73%
(USB)	Damaged USB Port	110	84%
(LB)	Battery replacement	90	93%
(O)	Overheating	70	100%

Total = 1000

i. Fill in the table's **cumulative percentage** column and construct the Pareto chart for this information. **2**

(1) Column graph in descending order

(1) Ascending line graph



ii. Which computer problems account for 70% of the inquiries? Give a reason for why this may be the case. **2**

(1) Screen, malfunctioning trackpad and malfunctioning keyboard account for 70%. These inquiries are related to the external components of the computer which are prone to damage and thus require more repairs. (1)

Examination continues overleaf...

Question 22 (4 marks)

Differentiate the following:

(a) $\ln(\tan 3x)$ **2**

$\Leftarrow \ln(f(x))$

$\frac{d}{dx}\left(\ln(\tan 3x)\right) = \frac{3\sec^2 3x}{\tan 3x}$ (2)

(b) $\frac{x}{e^{3x+1}}$ **2**

$u = x \quad u' = 1$

$v = e^{3x+1} \quad v' = 3e^{3x+1}$

$\frac{d}{dx}\left(\frac{x}{e^{3x+1}}\right) = \frac{e^{3x+1}(1) - x(3e^{3x+1})}{(e^{3x+1})^2}$ (1)

$= \frac{e^{3x+1}(1-3x)}{(e^{3x+1})^2}$

$= \frac{1-3x}{e^{3x+1}}$ (1)

Question 23 (3 marks)

Find the equation of the normal to the curve $y = \log_e(2x^2+1)$ at the point $(2, \log_e 9)$. **3**

$y' = \frac{4x}{2x^2+1}$, at $(2, \log_e 9)$ $y' = \frac{4(2)}{2(2^2)+1}$

$= \frac{8}{9}$

$m_N = \frac{-1}{m_T}$

$= \frac{-1}{\frac{8}{9}}$

$= -\frac{9}{8}$ (1)

$y - \log_e 9 = -\frac{9}{8}(x-2)$

$y - \log_e 9 = -\frac{9}{8}x + \frac{9}{4}$

$\therefore y = -\frac{9}{8}x + \log_e 9 + \frac{9}{4}$ (1)

* General form also acceptable $\Rightarrow 9x + 8y - 18 - 8\log_e 9$

Examination continues overleaf...

Question 24 (4 marks)

(a) Find: $\int (1+e^{7x})\,dx$ **2**

$= x + \frac{1}{7}e^{7x} + C$ (1) (1)

(b) Evaluate: $\int_0^2 \frac{2}{5x+4}dx = 2\int_0^2 \frac{1}{5x+4}\,dx$ **2**

$= \frac{2}{5}\int_0^2 \frac{5}{5x+4}\,dx$ (1)

$= \frac{2}{5}\left[\ln(5x+4)\right]_0^2$

$= \frac{2}{5}\left[\ln(14) - \ln(4)\right]$

$= \frac{2}{5}\ln\left(\frac{7}{2}\right)$ (1)

Question 25 (4 marks)

(a) Find $\frac{d}{dx}\left(xe^{2x}\right)$. **2**

$u = x \quad v = e^{2x}$

$u' = 1 \quad v' = 2e^{2x}$ (1)

$\frac{d}{dx}\left(xe^{2x}\right) = x(2e^{2x}) + e^{2x}(1)$

$= 2xe^{2x} + e^{2x}$ (1)

(b) Hence or otherwise, find $\int xe^{2x}\,dx$ **2**

$xe^{2x} + C = \int 2xe^{2x} + e^{2x}\,dx$

$2\int xe^{2x}\,dx = xe^{2x} - \int e^{2x}\,dx$ (1)

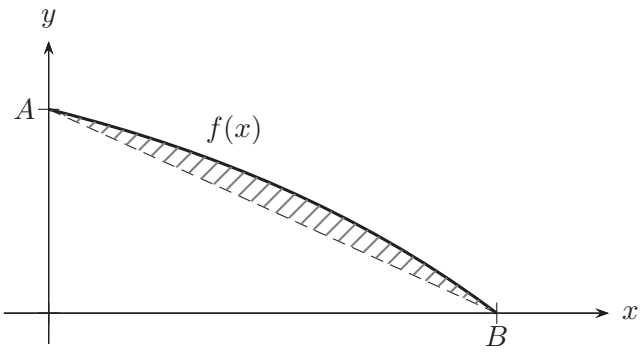
$= xe^{2x} - \frac{1}{2}e^{2x} + C$

$\int xe^{2x}\,dx = \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + \frac{C}{2}$ (1)

Examination continues overleaf...

Question 26 (3 marks)

Part of the graph of the function $f(x) = 9 - 3^{x+1}$ is shown below. **3**



$f(x)$ intersects the axes at the points A and B. The line joining A and B is also shown.

Show that the exact value of the shaded area is

$$6 - \frac{6}{\ln 3}$$

When $x=0$: $f(0) = 9-3$
$= 6$

When $y=0$: $0 = 9 - 3^{x+1}$
$3^{x+1} = 3^2$
$x+1 = 2$
$x = 1$

$\therefore A(0,6), B(1,0)$ (1)

$A = \int_0^1 9 - 3^{x+1}\,dx - \frac{1}{2}(1\times 6)$ (1)

$= \left[9x - \frac{3^{x+1}}{\ln 3}\right]_0^1 - 3$

$= \left[\left(9 - \frac{3^2}{\ln 3}\right) - \left(0 - \frac{3}{\ln 3}\right)\right] - 3$

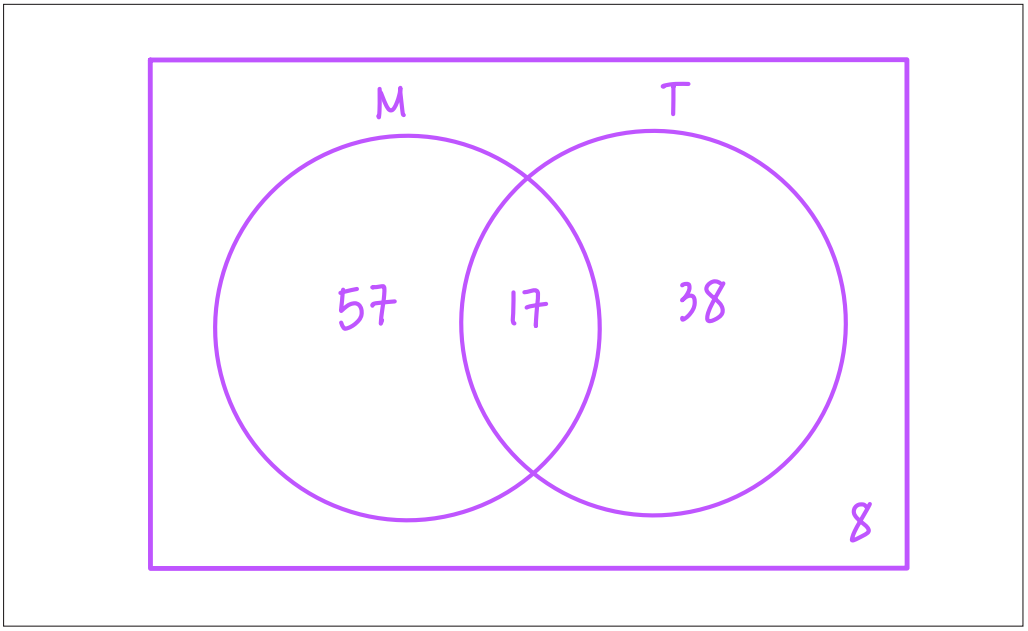
$= 9 - \frac{6}{\ln 3} - 3$

$= 6 - \frac{6}{\ln 3}$ (1)

Question 27 (2 marks)

A sports magazine surveyed 120 soccer fans and asked them whether they supported Manchester United or Tottenham. 74 fans supported Manchester United, 55 supported Tottenham and 8 supported neither team.

(a) Represent this data in a Venn diagram in the space below. **1**



(b) A fan is selected at random. If the fan selected is a Manchester United fan, what is the probability that they are also a Tottenham fan? **1**

$P(M \text{ and } T) = \frac{17}{74}$

Question 28 (5 marks)

Elysia and Leo are playing a game, they take turns throwing two dice. The game is won by the first player to throw a double six. Elysia starts the game.

(a) What is the probability that Elysia wins the game on the first throw. **1**

$P(\text{win first throw}) = \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{36}$

Examination continues overleaf...

(b) What is the probability that Elysia wins the game on the first or on the second throw? **2**

$P(\text{win on second throw}) = \frac{35}{36} \times \frac{35}{36} \times \frac{1}{36}$ ①

$P(\text{win first or second throw}) = \frac{1}{36} + \left(\frac{35}{36} \times \frac{35}{36} \times \frac{1}{36}\right)$

$= \frac{2521}{46656}$ ①

$= 0.054 \quad (2 \text{ dec. pl.})$

(c) Find the probability that Elysia eventually wins the game. **2**

$P(\text{win on third throw}) = \frac{35}{36} \times \frac{35}{36} \times \frac{35}{36} \times \frac{35}{36} \times \frac{1}{36}$

$\frac{1}{36} + \frac{1}{36}\left(\frac{35}{36}\right)^2 + \frac{1}{36}\left(\frac{35}{36}\right)^4 + \ldots$ ①

Forms a GP, $a = \frac{1}{36}$, $r = \left(\frac{35}{36}\right)^2$

$\therefore S_\infty = \frac{a}{1-r}$

$= \frac{\frac{1}{36}}{1 - \left(\frac{35}{36}\right)^2}$

$= \frac{36}{71}$ ①

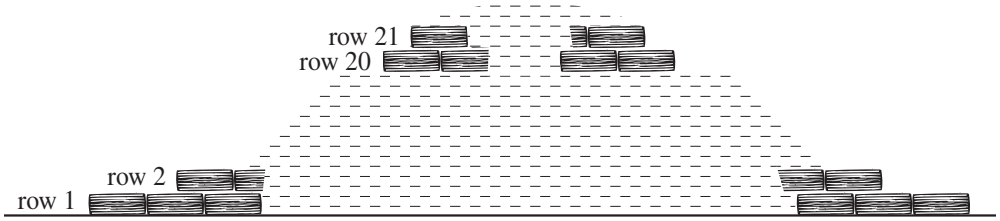
Examination continues overleaf...

Question 29 (5 marks)

Mr Park has built a new brick wall in his backyard to patch up the hole that had been created when a car ran into his fence.

The bottom row has 75 bricks, and each of the rows 2, 3, ..., 20 has 3 fewer bricks than the row below it.

Above row 20, each row has 1 brick fewer than the row below it. The top row has 10 bricks.



(a) How many bricks are in row 20? **2**

$a = 75,\ d = -3,\quad T_n = a + (n-1)d$

$T_{20} = 75 + (20-1)(-3)$ (1)

$= 18$ (1)

$\therefore$ 18 bricks in row 20

(b) What is the total number of rows in the wall? **1**

18 bricks in row 20, one less in each above row 20,

$18 + 10 = 28$

$\therefore$ 28 rows

(c) How many bricks are used in the construction of the wall? **2**

$S_n = \frac{n}{2}(a + l)$

$S_{20} = \frac{20}{2}(75 + 18)$

$= 930$

Row 21~28 : $17 + 16 + 15 + \ldots + 10$

$\therefore S_{21\sim28} = \frac{8}{2}(17 + 10)$

$= 108$ (1)

$930 + 108 = 1038$

$\therefore$ 1038 bricks (1)

Examination continues overleaf...

Question 30 (5 marks)

Scientists have recently discovered a new drug to control a rare medical condition. It is known that the quantity Q of the drug remaining in the body after t hours satisfies an equation of the form

$$Q = Q_0 e^{-kt}$$

where Q_0 and k are positive constants.

The initial dose is 5 milligrams and after 17 hours the amount remaining in the body is half the initial dose.

(a) Find the exact values of Q_0 and k. **3**

When $t=0$, $Q=5$mg

$5 = Q_0 e^{-k(0)}$

$\therefore Q_0 = 5$ (1)

When $t=17$, $Q = \frac{5}{2}$

$\frac{5}{2} = 5e^{-17k}$ (1)

$\frac{1}{2} = e^{-17k}$

$\ln\left(\frac{1}{2}\right) = \ln\left(e^{-17k}\right)$

$-17k = \ln\left(\frac{1}{2}\right)$

$\therefore k = -\frac{1}{17}\ln\left(\frac{1}{2}\right)$ (1)

(b) When will one-seventh of the initial dose remain? **2**

$Q = \frac{1}{7} \times 5$

(1) $\frac{5}{7} = 5e^{-kt}$ where $k = -\frac{1}{17}\ln\left(\frac{1}{2}\right)$

$e^{-kt} = \frac{1}{7}$

$-kt = \ln\left(\frac{1}{7}\right)$

$t = -\frac{1}{k}\ln\left(\frac{1}{7}\right)$

$= \frac{17}{\ln\left(\frac{1}{2}\right)} \times \ln\left(\frac{1}{7}\right)$

$= 47.7250\ldots$

$= 48$ (nearest hour) (1)

$\therefore$ 48 hours

Question 31 (5 marks)

A particle with velocity, v ms^{-1} moves along a straight line from a fixed point O

$$v = -4\cos t$$

where the time t is measured in seconds from $t = 0$.
It is initially 1 metre to the right of the origin.

(a) When, and where, does the particle first come to rest? **2**

Comes to rest when $v=0$:

$-4\cos t = 0$

$\cos t = 0$

$\therefore t = \frac{\pi}{2}, \frac{3\pi}{2}$ (1)

$x = \int -4\cos t \, dt$

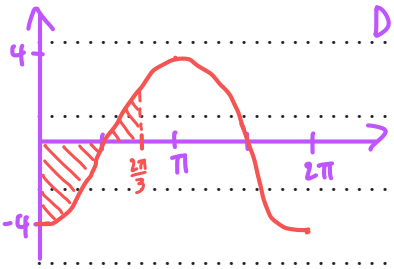
$= -4\sin t + C$, when $t=0, C=1$ (1)

$\therefore C = 1$

$x = 1 - 4\sin t$ $\quad \therefore x = -3, 5$

$\therefore$ Comes to rest when $t = \frac{\pi}{2}$, 3m to the left of the origin.

(b) Show that the total distance travelled by the particle between $0 \leq t \leq \frac{2\pi}{3}$ is $(8 - 2\sqrt{3})$ metres. **3**



$V = -4\cos t$

$a = 4, T = 2\pi$

(1)

$\text{Distance} = \left| \int_0^{\pi/2} -4\cos t \, dt \right| + \int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} -4\cos t \, dt$

$= \left| -4\left[\sin t\right]_0^{\pi/2} \right| - 4\left[\sin t\right]_{\frac{\pi}{2}}^{\frac{2\pi}{3}}$ (1)

$= \left| -4(1-0) \right| - 4\left(\frac{\sqrt{3}}{2} - 1\right)$ $\quad \frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3}$

$= 4 - 2\sqrt{3} + 4$

$= 8 - 2\sqrt{3}$ (1)

$\therefore$ Distance $= 8 - 2\sqrt{3}$ metres

Examination continues overleaf...

Question 32 (6 marks)

A ride sharing vehicle is to complete an interstate trip totalling 1000 kilometres at a constant speed of v km/h. When travelling at this speed the car consumes fuel at the rate of

$$\left(6+\frac{v^2}{50}\right) \text{ litres per hour.}$$

As a bonus, the company has offered to pay 50 cents per litre for fuel and for safety, two drivers where they will each be paid 45 dollars per hour whilst they are completing the trip.

(a) Let C represent the total cost of fuel and the drivers' wages for the trip. **3**

Show that

$$C = 10v + \frac{53000}{v} \quad \frac{93\,000}{v}$$

if the company was to pay two drivers.

· Fuel consumed at $\left(6+\frac{v^2}{50}\right)$ L/h, but fuel is \$0.50/L.

$\therefore C = \frac{1}{2}\left(6+\frac{v^2}{50}\right)$ \$/h

· Drivers' wages are \$90/h as it is \$45/h each.

· 1000km travelled at v km/h is done at $\frac{1000}{v}$ hours $\left(\frac{\text{km}}{\frac{\text{km}}{\text{h}}} = \text{h}\right)$

C = fuel + wages (1)

$= \frac{1}{2}\left(6+\frac{v^2}{50}\right) \times \left(\frac{1000}{v}\right) + 90\left(\frac{1000}{v}\right)$ (1)

$= \frac{500}{v}\left(6+\frac{v^2}{50}\right) + \frac{90\,000}{v}$

$= \frac{3000}{v} + 10v + \frac{90\,000}{v}$

$\therefore C = 10v + \frac{93\,000}{v}$ (1)

(b) The vehicle needs to complete the 1000 kilometre journey within 12 hours, whilst remaining under the speed limit of 90 km/h. **3**

At what speed v should the vehicle be driven at to minimise the cost C, whilst arriving at the destination within 12 hours?

$C = 10v + 93000v^{-1}$

$\frac{dC}{dv} = 10 - 93000v^{-2}$

$0 = 10 - \frac{93000}{v^2}$

$93000 = 10v^2$

$v^2 = \frac{93000}{10}$

$v = \pm\sqrt{9300}$

$\therefore v = \pm 10\sqrt{93}$

$\therefore v = 10\sqrt{93}$ km/h

$\approx 96.4365\ldots$

$= 96.14$ km/h

(2 dec. pl.)

$\frac{d^2C}{dv^2} = \frac{186\,000}{v^3}$

$\left.\frac{d^2C}{dv^2}\right|_{v=10\sqrt{93}} = \frac{186\,000}{(10\sqrt{93})^3}$

$= 0.20739\ldots > 0 \therefore$ min

$\left.\frac{d^2C}{dv^2}\right|_{v=-10\sqrt{93}} = \frac{186\,000}{(-10\sqrt{93})^3}$

$= -0.20739\ldots < 0 \therefore$ max

$\therefore t = \frac{1000}{10\sqrt{93}}$

$= 10.3695\ldots$

$= 10.37$ hours

However, if the journey takes 10.37 hours, drivers will exceed the speed limit.

$12 = \frac{1000}{V} \Rightarrow V = \frac{1000}{12}$

$= 83.33$ km/h

If the journey takes exactly 12 hours, drivers can stick to $V = 83.33$ km/h. Hence, if they drive at 90km/h,

$V = \frac{1000}{90}$

$= 11.1111\ldots$

$= 11.11$ hours

$\therefore$ Drivers can take 11.11 hours whilst driving at 90km/h.

Question 33 (5 marks)

For the function $f(x) = x^3 - 3x^2 - 9x + 18$, find the stationary points, point(s) of inflection and determine the nature of the stationary points. Then sketch the graph of $f(x)$, showing the turning points and point(s) of inflexion. **5**

$f'(x) = 3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$3(x-3)(x+1) = 0$

$\therefore x = -1, 3$

$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 18$

$= 23$

$f(3) = 3^3 - 3(3^2) - 9(3) + 18$

$= -9$

(1)

$f''(x) = 6x - 6$

$f''(-1) = -6 - 6$

$= -12 < 0$

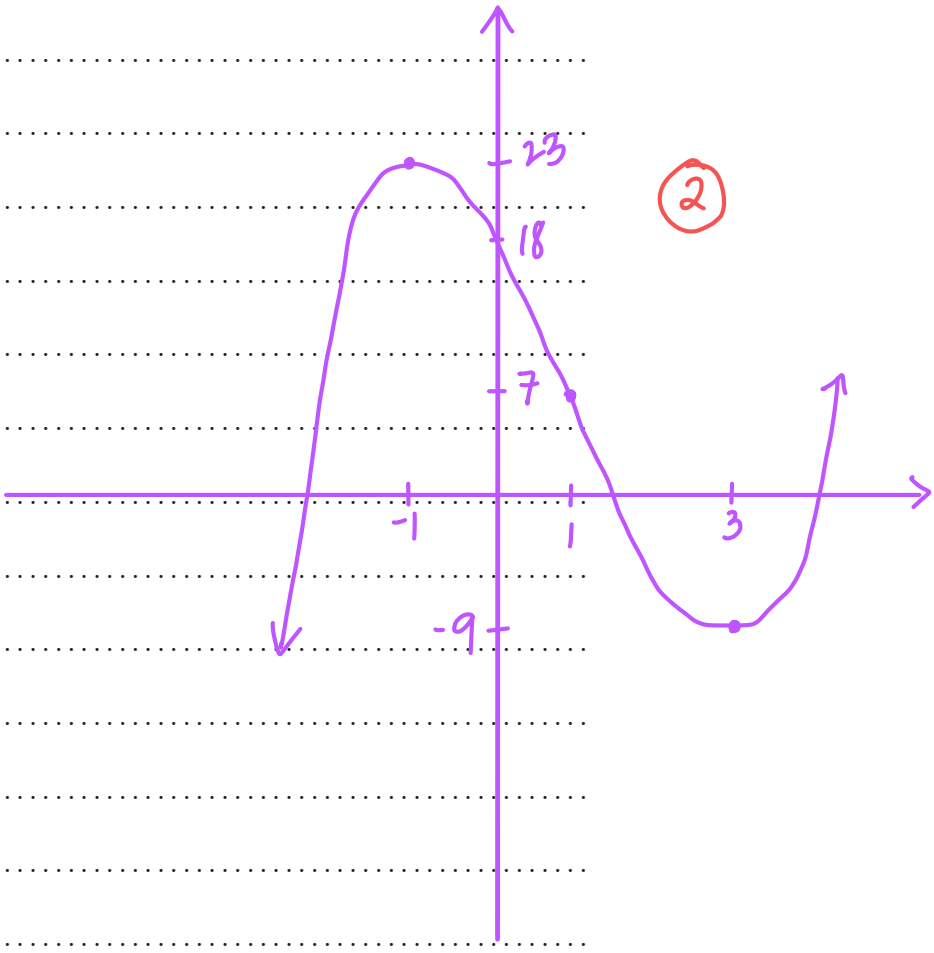
$\therefore (-1, 23)$ max

(1)

$f''(3) = 18 - 6$

$= 12 > 0$

$\therefore (3, -9)$ min



(2)

$f''(x) = 6(x-1) = 0$

$\therefore x = 1$

$f(1) = 1 - 3 - 9 + 18$

$= 7$

(1)

x	0	1	2
$f''(x)$	-6	0	6

As concavity changes $(1, 7)$ is a point of intersection.

End of paper.